Modeling, Control, and Simulation of a Single-Stage Capacitor Microphone

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Abstract—**In this work, a single-stage capacitor microphone which is controlled by a linear-quadratic regulator is presented. The microphone capacitor is composed of an electrical part, represented by a linear network composed by a resistor and a voltage source, and a mechanical part, represented by a capacitor with a movable plate, fixed to a wall by means of a resistor element. Finally, the system is coupled to the electrical network through a sliding surface element.**

The mathematical modeling of the system is presented, providing a set of nonlinear equations, which are then linearized around their equilibrium points. Finally, the linear-quadratic regulator is implemented by obtaining its control law and then simulating its behavior. Through the simulation results, the proper operation is verified.

Keywords—Capacitor microphone, linear-quadratic regulator, modelling, state-space model.

I. INTRODUCTION

An electromechanical device is, by definition, a system that combines Maxwell's electromagnetic theory with Newton's rational mechanics. Before the middle of the 18th century, studies of electromechanical systems were treated separately, as two isolated systems. However, with the progress and refinement of Maxwell's electromagnetic theory and the increasingly improved Newton's equations, the two theories could be coupled, resulting in a set of dynamic equations that model the electromechanical device as a single set, a single dynamic model [1].

In general, this model is composed of electrical and mechanical dynamic equations, coupled by some magnetic means. It is common to find that this magnetic coupling takes the form of inductances that are functions of parameters and gradients [2].

Within the many electromechanical devices that have been studied in the literature, it is very common to find those that develop a torque due to their rotating nature. Basically, these rotating devices are part of the conceptual stages that lead, finally, to the study and design of the electrical machines known today. However, electromechanical models are also found, but they develop forces instead of torques. This indicates, then, that they are electromechanical mechanisms of linear displacement. These systems are not frequently studied, but a number of applications can be found, such as contactors and electrical relays, dishwashers, longitudinal hydraulic presses, among others.

This study presents a single-stage capacitor microphone (SCM) which is modeled by obtaining its dynamic equations. Then a proposed control system is presented, to finally simulate the device, verifying the model through the analysis of the simulation results.

II. TOPOLOGY OF THE SCM

The topology presented in Fig. 1, was proposed by [3]. From Fig. 1 it can be seen that the SCM is composed of a simple electrical network comprising a supply voltage $v(t)$ connected to a resistor R and the circuit is closed by a movable plate capacitor and a sliding element corresponding to the negative pole of the circuit. A current $i(t)$ flows through the electrical circuit. The capacitor is built by two plates of fixed area A and separated by a variable distance $x(t)$. Between the two plates there is an insulating medium of permeability ε. The minimum distance between the two plates corresponds to l_o. The movable plate is connected to the fixed lateral surface by means of the spring of constant k. On the other hand, the sliding element has a damping coefficient b.

All components mentioned in the system are assumed to be linear.

III. MODELING OF THE SCM

Assuming that the capacitor stores an electric charge $q(t)$ and according to Fig. 1, the capacitance C is a function of the distance $x(t)$, i.e. $C(x(t))$. Therefore, the energy stored in the capacitor can be expressed as

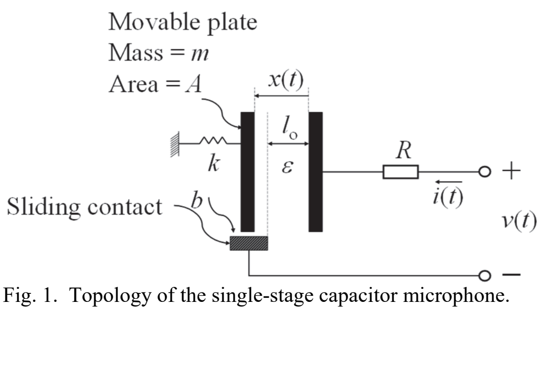

Fig. 1. Topology of the single-stage capacitor microphone.

$$W_e(x(t)) = \frac{1}{2} \cdot \frac{q(t)^2}{C(x(t))}. \quad (1)$$

The mechanical power expression [1] associated with the capacitor is given by

$$\begin{gathered} \frac{\mathrm{d}W_e(x(t))}{\mathrm{d}t} = -\frac{\partial W_e(x(t))}{\partial x(t)} \cdot \frac{\mathrm{d}x(t)}{\mathrm{d}t} \Rightarrow \\ \Rightarrow \frac{\mathrm{d}W_e(x(t))}{\mathrm{d}t} = -\frac{1}{2} \cdot \frac{1}{\epsilon \cdot A} \cdot q(t)^2 \cdot \frac{\mathrm{d}x(t)}{\mathrm{d}t}. \end{gathered} \quad (2)$$

It is known that the mechanical power of a system is the linear product of the force experienced by that system times the velocity of the system. Therefore, by inspection of (2), it is concluded that the expression for the force [1] is given by

$$f_e(t) = -\frac{1}{2} \cdot \frac{1}{\epsilon \cdot A} \cdot q(t)^2. \quad (3)$$

Note that the minus sign in (3) is due to the fact that the force $f_e(t)$ tends to decrease the separation of the plates of C. According to Fig. 1, the dynamic equations relating the electrical and mechanical system are presented in equations (4) and (5) respectively.

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = \frac{1}{R} \cdot \left(-\frac{1}{\epsilon \cdot A} \cdot x(t) \cdot q(t) + v(t)\right) \quad (4)$$

$$\begin{gathered} \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = \frac{1}{m} \cdot \Big(b \cdot \frac{\mathrm{d}x(t)}{\mathrm{d}t} - \\ -k \cdot (x(t) - l_o) - \frac{1}{2} \cdot \frac{1}{\epsilon \cdot A} \cdot q(t)^2\Big). \end{gathered} \quad (5)$$

Equations (4) and (5) represent the SCM model which are nonlinear equations. Since one of the objectives of the study is to design a linear control system for the SCM, the model in (4) and (5) is linearized by using the perturbation technique and the Taylor series expansion [4], [5], depending on the SCM operating points in steady state (EPs). Therefore, the EPs are obtained by zeroing the derivatives. The steady state SCM model is obtained from this action and is presented in (6).

$$\begin{cases} X \cdot Q = \varepsilon \cdot A \cdot V \\ k \cdot (X - l_o) = -\frac{1}{2} \cdot \frac{Q^2}{\epsilon \cdot A}, \end{cases} \quad (6)$$

where each of the variables in capital letters stand for the variables in steady state.

Finally, linearizing the model in (4) and (5) and considering (6), the linear small-signal model is obtained and presented in (7).

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot u \\ \mathbf{y} = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot u. \end{cases} \quad (7)$$

From the model in (7), the state variables, input signal and output signals are defined as $\mathbf{x} = [q(t), x(t), \mathrm{d}x(t)/\mathrm{d}t]^{\mathrm{T}}$, $u = v(t)$, and $\mathbf{y} = \mathbf{x}$ respectively. Moreover, $\{\mathbf{x}, \mathbf{y}\} \in \{\Re\}^3$. On the other hand, the matrices of the model in (7) are defined in (8).

$$\mathbf{A} = \begin{bmatrix} -\frac{X}{\epsilon \cdot A \cdot R} & -\frac{Q}{\epsilon \cdot A \cdot R} & 0 \\ 0 & 0 & 1 \\ -\frac{Q}{\epsilon \cdot A \cdot m} & -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{R} & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C} = \mathbf{I}_{3x3}$$

$$\mathbf{D} = \mathbf{0}_{3x1}, \quad (8)$$

where $\{\mathbf{A}, \mathbf{C}\}$ $\mathbb{M}_{3x3}\{\Re\}$ and $\{\mathbf{B}, \mathbf{D}\}$ $\mathbb{M}_{3x1}\{\Re\}$. From the equation in (7), the hatted variables are assumed to be much smaller than their steady-state variables.

Finally, equations (7) and (8) represent the small-signal linear state-space model of the SCM.

IV. Control System of the SCM

When obtaining a linear model of the SCM with a single input represented in state-space, a candidate control system is a linear quadratic regulator (LQR) [4], [6] and that for this particular application, to be the most suitable alternative. Therefore, the chosen control system is an LQR.

It is desired to control the displacement $x(t)$, thereby varying the SCM capacitance as a function of the voltage regulation $v(t)$. It will be assumed that another device external to the SCM is in charge of regulating this voltage, but getting the actuating signal from the LQR. According to this regulator, the control law is defined as $u = -\mathbf{K} \cdot \mathbf{x}$. The cost function to be minimized [6] is defined in (9).

$$J = \int_0^\infty (\mathbf{x}^\mathbf{T} \cdot \mathbf{Q} \cdot \mathbf{x} + \mathbf{u}^\mathbf{T} \cdot \mathbf{R} \cdot \mathbf{u}) \cdot \mathrm{d}t, \quad (9)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are defined as positive hermitic or real symmetric matrices. Overall, the function J is a cost function of the energy related to the states ($\mathbf{x}$) and control signals ($\mathbf{u}$) of the model concerned, and therefore, $\mathbf{Q}$ and $\mathbf{R}$ define the relative significance of the error and cost of this energy [4], [6]. In this study $\mathbf{Q}$ and $\mathbf{R}$ are given as

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = [0.01]. \quad (10)$$

Note that in this case, R is a constant since u is a scalar function, defined above. Since the matrices $\mathbf{A}$,

TABLE I. SCM PARAMETERS

Parameters	Values
ε	1 V/m
A	$4.5\cdot10^{-5}$ m^2
l_o	0.005 m
k	100 N/m
b	0.1 kg/s
V	24 V
R	1 mΩ
m	0.01 kg

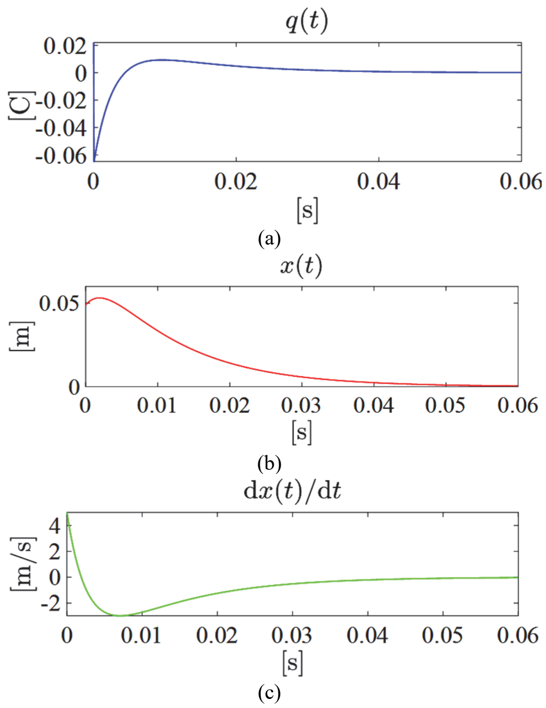

Fig. 2. Step response curves considering the initial conditions vector $\mathbf{x_0}$. (a) Step response regarding $q(t)$. (b) Step response regarding $x(t)$. (c) Step response regarding d$x(t)$/dt.

B, **Q**, and the constant R are known, it is possible to calculate **K** through the use of the MATLAB command *lqr* [4]. The values of $\mathbf{K} = 10^3\cdot[2.1825\ 0.9791\ 0.0098]$. Replacing the control law (u) in (7), the optimal model of the SCM is obtained which is defined in (11).

$$\begin{cases}\dot{\mathbf{x}} = \mathbf{AA}\cdot\mathbf{x} + \mathbf{BB}\cdot u \\ \mathbf{y} = \mathbf{CC}\cdot\mathbf{x} + \mathbf{DD}\cdot u,\end{cases} \tag{11}$$

where $\mathbf{AA} = \mathbf{A} - \mathbf{B}\cdot\mathbf{K}$, and $\mathbf{BB} = \mathbf{CC} = \mathbf{DD} = \mathbf{I}_{3x3}$, and **I** is the identity matrix of 3x3-size.

V. SIMULATION RESULTS

The simulation developed is static and was carried out in MATLAB. The simulation parameters are listed in Table I. On the other hand, initial conditions equivalent to the values of the variables in steady state are assumed, i.e., the initial state vector $\mathbf{x_0} = [Q, X, 0]^T \Rightarrow \mathbf{x_0} = [0.022, 0.049, 0]^T$.

Fig. 2 shows the simulation results for the state variables $q(t)$ (see Fig. 2(a)), $x(t)$ (see Fig. 2(b)), and d$x(t)$/dt (see Fig. 2(c)), versus a step change, considering their initial values at $\mathbf{x_0}$. In all the responses described in Fig. 2, it can be observed that they reach their initial values at x0. From Fig. 2, it can be observed that they reach their steady state values (remember that, as this is an LQR system, its steady state values must be zero [4]) fairly quickly, i.e., already at 50 ms, the system has reached its steady state regime.

In summary, it can be seen that the designed LQR operates properly with the SCM, which has no overshoots, provides fast dynamics with zero steady-state error.

VI. CONCLUSION

A microphone capacitor controlled by an LQR has been presented. In addition, its operation has been verified through a static simulation. From the simulation results, the proper operation of each of the state variables, which were plotted, has been verified. None of the simulated variables have overshoots, showing zero steady state errors and with a remarkable speed in reaching their steady state values.

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